



Teacher Guide for the Black Hole Orbits Instant Pack

In this Teen Astronomy Cafe – To Go! Instant Pack, students review [Newton's laws of gravity](#) and learn the basics needed to understand numerical integration before applying it to the [motion of a particle around a black hole](#). Students [apply numerical integration](#) to simulate a ball at a constant velocity, a ball with changing velocity, and a ball on a spring. With this information, students [compare the general theory of relativity](#) to [Newton's law of universal gravitation](#) and [evaluate](#) which theory would be suited to determining a [particle's motion around a black hole](#), and explain why. This activity is conducted in a Python Notebook, a web-based interactive computational environment that contains code, text, and plots.

Learning Objectives:

Students will be able to:

- use numerical integration to plot and analyze the motion of a ball at constant velocity, changing velocity, and on a spring.
- compare and contrast Newton's law of gravity with the gravitational law of general relativity by analyzing a simulation of a particle orbiting a black hole.

NGSS Standards:

Building Towards NGSS Performance Expectations:

- HS-PS2-1: Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
- HS-PS2-4: Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

Disciplinary Core Ideas	<p>HS-PS2.A: Forces and Motion Newton's second law accurately predicts changes in the motion of macroscopic objects.</p> <p>HS-PS2.B: Types of Interactions Newton's law of universal gravitation provides the mathematical model to describe and predict the effects of gravitational forces between distant objects.</p> <p>Forces at a distance are explained by fields (gravitational) permeating space that can transfer energy through space.</p>
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<p>Science and Engineering Practices</p>	<p>Using Mathematics and Computational Thinking Students use mathematical, computational, and algorithmic representations to numerically integrate Newton's Laws and explore the motion of a particle around a black hole.</p> <p>Analyzing and Interpreting Data Students analyze data using mathematical and computational models in order to compare and contrast Newton's law of gravity with the gravitational law of general relativity.</p> <p>Developing and Using Models Students evaluate the merits and limitations of two different models of Newtonian and Einsteinian orbits in order to select the model that best fits the evidence of particle motion around a black hole.</p> <p>Connections to Nature of Science: Science Models, Laws, Mechanisms, and Theories Explain Natural Phenomena Students determine if Newton's law of gravity or the gravitational law of general relativity is better suited to determining a particle's orbital motion around a black hole.</p>
<p>Cross Cutting Concepts</p>	<p>Systems and System Models Mathematical and computational models are used to simulate black hole systems and interactions with particles, leading to the evaluation of our understanding of gravity.</p> <p>Connections to the Nature of Science: Science Is a Way of Knowing Students compare the historical evolution of gravitational laws (Newton's law of gravitation and Einstein's general theory of relativity) to best determine the motion of a particle around a black hole.</p>

Suggested Timing:

- Black Hole Orbits Phenomenon & Presentation (50 minutes)
- Python Notebook intro and how to run a cell (5 minutes)
- Activity 1: Newton's Law of Universal Gravitation (15 minutes)
- Activity 2: Introduction to Numerical Integration (20 minutes)
- Activity 3: The Gravity We Know: Newton's Law, Integrated (15 - 20 minutes)
- Activity 4: The Gravity Near a Black Hole: Orbits in General Relativity (20 minutes)

Black Hole Orbits Presentation:

This Teen Astronomy Café To — Go! Instant Pack includes a slideshow presentation that is accompanied by a recorded presentation video. The purpose of the slideshow is to provide the audience with background information on black hole orbits and to excite and motivate learners in the content. A lesson-level phenomenon (below) is suggested to precede the presentation slides and is recommended as a student-driven active learning strategy to initiate curiosity and questioning. Instructions for incorporating the phenomenon are included below.



Black Hole Orbits Phenomenon:

Simulate a Black Hole!

Materials:

- Large circular hoop (embroidery hoop, hula hoop, etc.)
- Spandex fabric – enough to stretch across the hoop
- 8–16 binder clips to hold the fabric in place along the outside edge of the hoop
- A mass to create the well, such as a 2lb toning ball
- Balls of different mass (marbles, ping pong balls, steel balls)

Instructions

Set-up

1. First, arrange a Driving Question Board with the lesson question: “How does gravity, especially of a black hole, affect the motion of a nearby object?” During this phenomenon activity, students will demonstrate how a large mass such as a black hole curves spacetime and what effect this has on other nearby objects. The Python Notebook activity will take a closer look at comparing Newton's law of gravity to Einstein's general theory of relativity.
2. To construct the black hole simulator pull the fabric tight and flat over the hoop. Secure the fabric using the binder clips. The fabric represents spacetime.

Observe NASA's Black Hole Orrery

3. Before experimenting with the black hole simulator, show the class [NASA's Black Hole Orrery](#).
4. After viewing the orrery, visit the Driving Question Board and the driving question: “How does gravity, especially of a black hole, affect the motion of a nearby object?” Students should develop questions based on the orrery and the driving question.
5. Students should share their generated questions as a whole class by posting them visibly to the Driving Question Board. The questions can be organized into similar categories and revisited while experimenting with the black hole simulator, slideshow presentation, and/or Python Notebook.

Black Hole Simulator

6. Gather students around the black hole simulator. They can help to make sure the fabric stays pulled tight.
7. Place a marble on the fabric. Observe.
8. Pick up the marble and try to roll it across the fabric. Observe the shape of the path of the marble.
9. Place the heavy round object (2lb toning ball) in the center of the fabric. Observe what happens to the fabric.
10. Ask students to predict the marble's path if a heavy object was placed in the center of the fabric.
11. Next, place the marble along the edge of the fabric and observe what happens to the marble.
12. Pick up the marble and try to roll it across the fabric. Observe what happens. Have students describe the marble's path.
13. Have students continue to experiment with more marbles, different speeds, and balls of different mass. Observe and compare the paths of the marbles or balls with these different experiments.
14. Concluding experiments, have students return to the Driving Question Board and answer, revise, or add to their questions.
15. Begin the presentation and Python Notebook activity. Throughout the activity, the whole class should refer back to the Driving Question Board to answer, refine or ask new questions.

1 Activity adapted from: <https://www.scientificamerican.com/article/make-a-black-hole/> and <https://jesford.github.io/materials/FordStangAnderson15.pdf>



Python Notebook General Information:

- Start by going over the operation of a Python Notebook: To execute or run a selected cell, click the little play button or hit [Shift + Enter] on your keyboard. Some cells may take a few seconds to render, so be patient!
- If something doesn't seem to be working correctly (e.g., it can't find resources such as tools.ipynb, or the first simulation where students don't have to enter in any values fails), try restarting the notebook (Runtime → Restart).

Activity 1: Newton's Law Of Universal Gravitation

This activity goes more smoothly if students understand the definitions of velocity and acceleration. Go over these definitions as needed depending on the audience...and remember they are both vectors!

The acceleration due to gravity on a particle due to another object can be found using the equation $a(r) = -\frac{Gm}{r^2}$. The negative sign indicates the force is attractive. G is the gravitational constant, m is the mass of the object and r is the distance between the objects. Note that the force depends on $1/r^2$. Doubling the distance between objects decreases the force by a factor of four.

Particles have a velocity as well. Velocity and acceleration do not have to be in the same direction. If you throw a ball up in the air, its velocity points upward but the acceleration due to gravity is downward.

The first cell the students run defines a function that calculates the acceleration due to gravity. The mass of the black hole is input in solar masses (1 solar mass is the mass of our Sun) and the distance between the objects is entered in astronomical units or AU (one AU is the distance from Earth to the Sun).

The next cell calls that function plugging in the mass of the black hole (4.3×10^6 solar masses) and the distance of 1 AU. Students can use this function to explore the acceleration of situations by changing the mass and distance in this cell and running it again. This is done in the third cell where the mass is changed to 1 (representing 1 solar mass) and 1 AU. Students can see the acceleration due to our Sun is much less than that due to the black hole at the center of our galaxy given the same distance!

Activity 2: Introduction to Numerical Integration

Activity 2.1: To Start With

The simplest case to explore is that in which the velocity is constant (and the acceleration is zero). Most students are probably familiar with this situation.

The first cell defines a function called ConstantBall. When you input the velocity and time, ConstantBall simply multiplies the two and returns the result, which is the distance traveled.

The second cell lets students input various values for velocity and time and observe the result. Be sure the students run the cell each time they input new values to see the result!



Activity 2.2: Ball With Changing Velocity

This activity looks at a ball rolling down a hill. The velocity of the ball changes as it rolls down the hill. Although some students who have taken physics might be able to solve this problem exactly (*by using* $x(t) = v_0t + \frac{1}{2}at^2$), this activity introduces the concept of numerical integration.

In numerical integration, you divide the time up into small intervals and assume the velocity is constant during each of those intervals. You then add up all those little distances and you find a very close approximation to the total distance traveled. The shorter the time intervals you use, the more accurate the approximation becomes.

The cells define the velocity function as $v(x) = x$ and then perform numerical integration to graph position vs time. Students can experiment with different starting positions and different numbers of integrations and observe how changing these values changes the shape of the graph.

```

[12] x0 = FloatSlider(min=-1, max=1, step=0.1, value=0.1)
      N  = IntSlider(min=1000, max=100000, step=1000, value = 100000)#, disabled=True)

interactive(MyFirstIntegrator, x0=x0, T=fixed(0.0001), N=N)
    
```

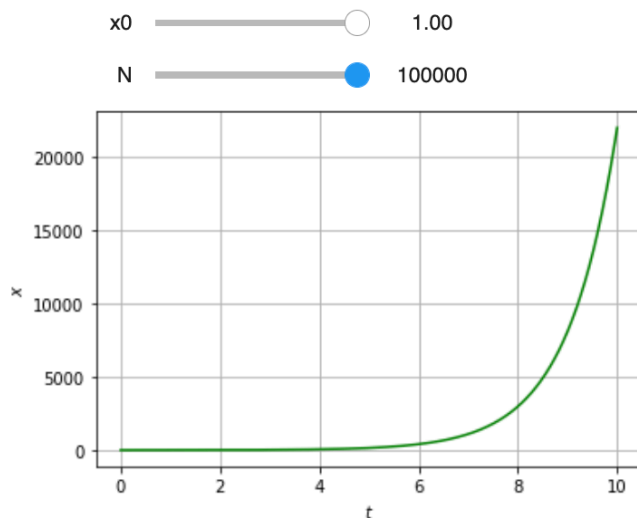


Figure 1: Graph showing position vs time in Activity 2.2.

Activity 2.3: Ball On A Spring

This activity shows some of the limitations of numerical integration by applying the technique to a ball on a spring (a simple harmonic oscillator). Unlike the previous activity, all of the code necessary to compute the position of the ball at any given time and display the results are contained in one cell. The code computes the velocity at any given position and adds that to the previous velocity to find the velocity at any given time. That velocity is then used to calculate the new position. Students can change the initial position, the initial velocity and the number of integrations.



For **very** small numbers of integrations, it may appear that this technique works. As you increase the number of integrations, something goes drastically wrong. If you pull a ball on a spring and release it, the ball should move back and forth (ignoring energy loss due to friction). In order to increase the amplitude of the motion, you would need to add energy. However, in this case, the amplitude of the oscillation increases with no apparent input of energy! Numerical integration failed in this case. Sometimes you have to make sure your results make sense!

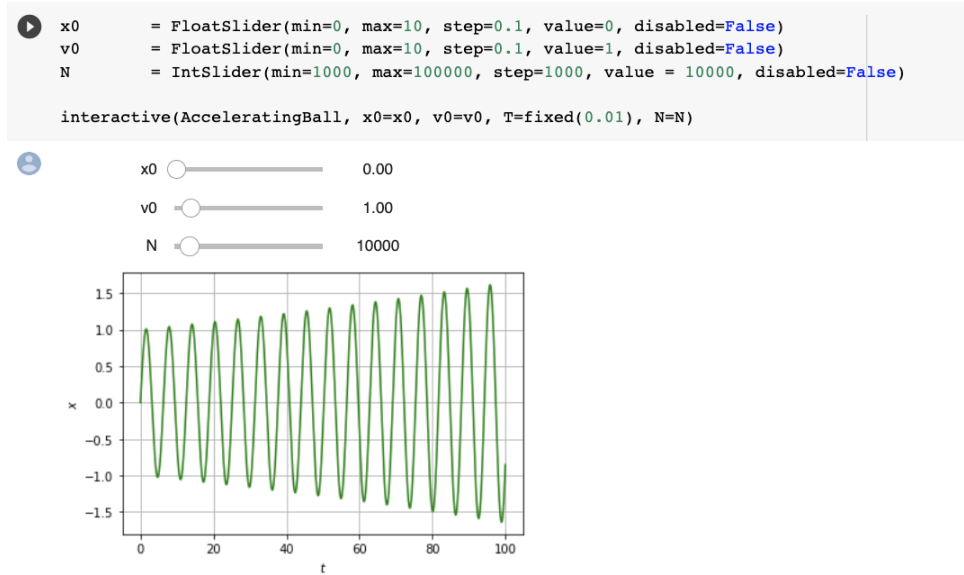


Figure 2: Graph representing a ball on a spring for Activity 2.3.

Activity 3: The Gravity We Know: Newton's Law, Integrated

This activity applies the numerical integration technique to Newton's law of gravitation from Activity 1. The code looks more complicated since you have to calculate the velocity and acceleration in two dimensions (x and y) instead of just one.

Students can change the sliders and observe similar effects to those in the previous activity: the object frequently gets ejected from the system, indicating energy is not conserved again!

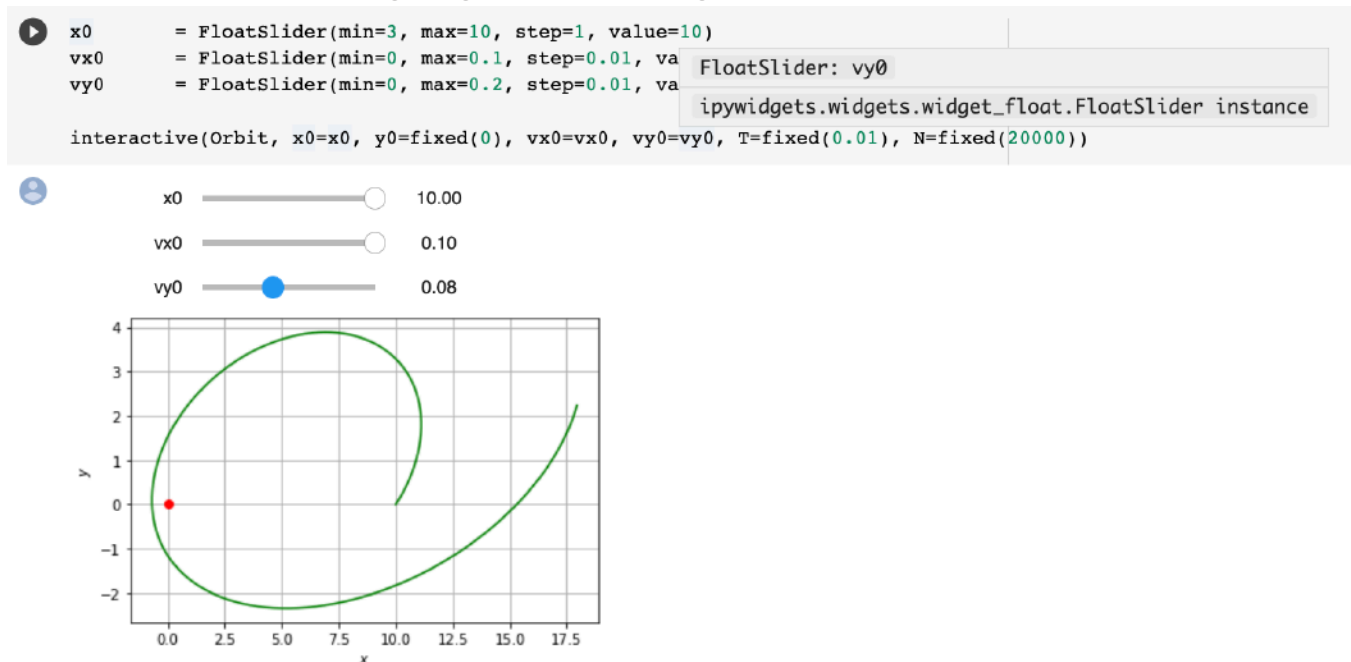


Figure 3: Students apply numerical intergration to Newton's law of gravitation in Activity 3.



Activity 4: The Gravity Near A Black Hole: Orbits In General Relativity

General relativity makes a small but important correction to Newton's law of gravitation. This correction is so small that it is not always important to apply it (Apollo astronauts made it to the Moon using Newtonian gravitation) but is more important in extreme situations such as near a black hole.

These cells create an orbit calculator similar to the one in Activity 3. There is a small checkbox added to the graph that lets students toggle between Newtonian gravity and general relativity. Students can experiment with the sliders and the toggle to see when it is important to make the correction and when it can be safely ignored.

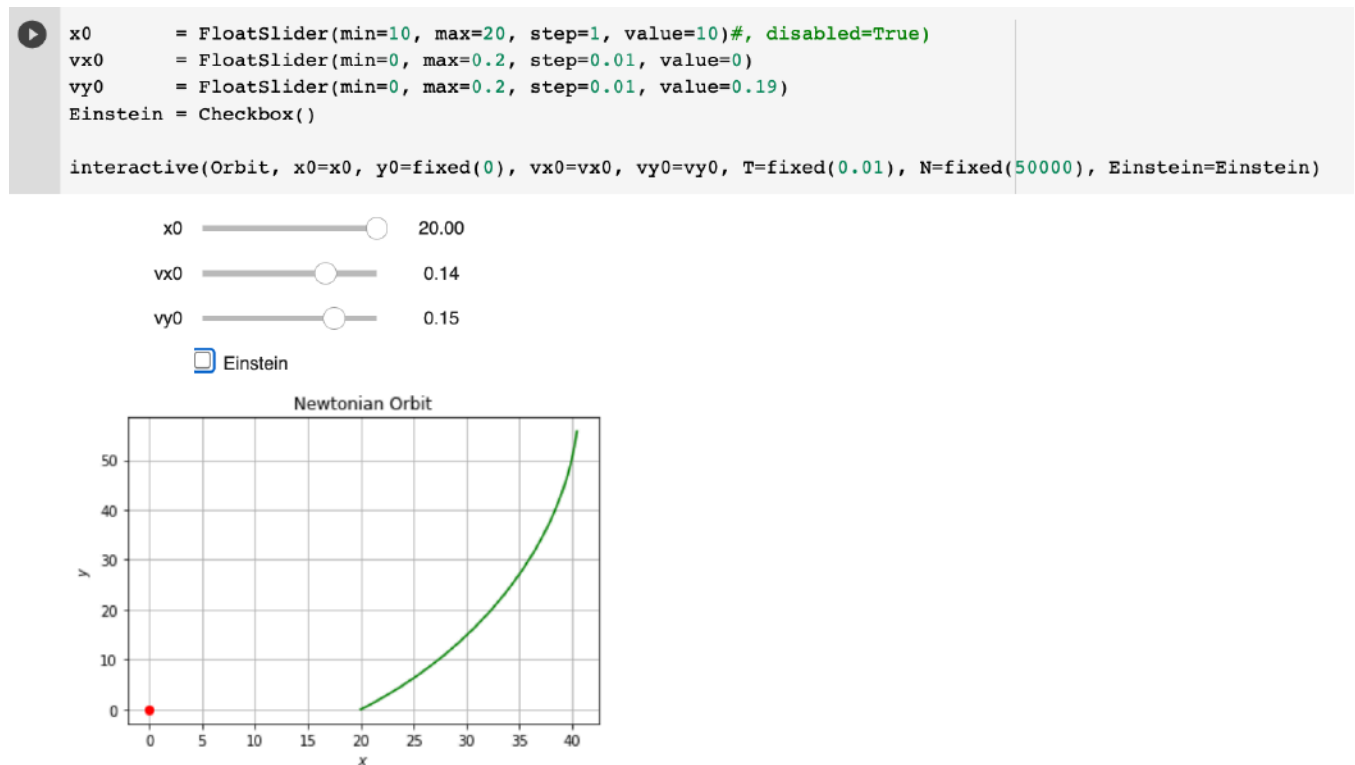


Figure 4: Students compare Newtonian gravity and general relativity in Activity 4.